**Institute of Cybernetics at Tallinn Technical University**
**Tartu University**
**Estonian Marine Institute**
**Centre of Biomedical Engineering of Tallinn Technical University**

# CENS

## Centre for Nonlinear Studies

**Annual Report**

**1999**

**Tallinn, Estonia**

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**Lühikokkuvõte**

Lainelevi analüüs mikrostruktuuriga materjalides on tuvastanud üksiklainete (solitonide) kujunemise tingimused normaalse ja anomaalse dispersiooni korral. On tuletatud termodünaamiliselt korrektne meetod 2D lainelevi simuleerimiseks diskreetsetest osadest koosnevas materjalis eesmärgiga uurida faasifrontide levi. On koostatud matemaatiline mudel, mis arvestab dissipatsiooni erinevust makro- ja mikrostruktuuris. Geomeetriast tingitud dispersiooni arvestamine on lubanud lahendada pöördülesande üksiklaine amplituudi ja kiiruse leidmiseks teades laineharjade mustrit (madala vee ülesanne). Bioloogiliste kudede mikrostruktuuri iseärasused on arvestatud südamelihase kontraktsiooni mudelis, mis on heas kooskõlas katsetega. On koostatud regulatsioonimehhanismi mudel, mis kirjeldab makroenergeetiliste fosforiühendite tootmist südamelihase rakus. On esitatud südamerütmi variaabelsuse fraktaalne analüüs Zipfi-tüüpi seaduse tuvastamisega. Mittelineaarsete filtrite kasutamine bioelektriliste signaalide (EEG, EKG) analüüsil on võimaldanud saada huvitavaid tulemusi nende interpreteerimisel. Pingeanalüüsi tarbeks on välja töötatud fotoelastsuse interferentsiribade teooria ja on loodud meetod jääkpingete määramiseks kihilise struktuuriga klaaskiududes. Tulemusi on rakendatud jääkpingete mõõtmiseks joogiklaasides, kineskoobi kaela torudes, optilistes kiududes ja paksudes plaatides. Arendatud on akustodiagnostika meetodeid mittehomogeense eelpingeolukorra määramiseks. On konstrueeritud seade klaverihaamrite parameetrite määramiseks. On kirjeldatud meetod diferentsiaalvõrrandite sümmeetria uurimiseks Cartani meetodil. On konstrueeritud ja uuritud gradueeritud q-diferentsiaalalgebra redutseeritud kvanttasandil. On näidatud Rossby lainete mõju tsonaalsele veetranspordile. On tõestatud Rossby lainete tasakaalustatud spektrite olemasolu ja stabiilsus. Eksperimentaalselt on näidatud dipoolkeeriste interaktsioonivõimalus ja anomaalsete keeriste olemasolu.

**Abstract**

The conditions of emergence of solitary waves (solitons) have been established for waves in microstructured materials at normal and anomalous dispersion. A thermodynamically consistent method is derived for analysing the 2D waves in materials including discrete elements. It is shown how dissipation at macro- and microstructure influences the wave parameters. The 2D surface solitons in shallow water are analysed and the inverse problem solved provided the wave pattern is known. A mathematical model for heart muscle contraction is derived and tested that takes the microstructure of the tissue into account. It is shown how intracellular metabolites regulate oxidative phosphorylation. The fractal analysis of heart rate variability (HRV) is carried out and new Zipf's law - based multiscaling behaviour of the HRV is discovered. The application of different nonlinear filters for the analysis of bioelectrical signals (ECG, EEG) has provided interesting results in their interpretation. The structure of the isochromatic fringe pattern of a photoelastic model is analysed and a method for complete stress measurement in step-index optical fibres and fibre preforms has been developed. Stress analysis is carried out for many practical cases using the methods of photoelasticity. The methods of acoustodiagnostics of material properties have been enhanced. A device for measuring the physical properties of piano hammers has been constructed. The symmetries of differential equations have been studied with the Cartan method. A generalization of graded q-differential algebra is proposed. It is shown that Rossby waves may influence the zonal transport of water. The existence and stability of thermodynamically equilibrated spectra were proved. Experimentally a new class of interaction of vortex dipols has been shown, as well as the anomalous pairing of small cyclons.

**Keywords: nonlinear dynamics, cardiac contraction, cell energetics, solitons, acoustodiagnostics, photoelasticity, Rossby waves, symmetries of differential equations, signal processing.**

# 1. Introduction

Nonlinear phenomena are encountered in many disciplines notably in mechanics and physics, mathematics, engineering sciences and biology. In Estonia, Institute of Cybernetics at Tallinn Technical University (IOC at TTU), Tartu University (TU), Estonian Marine Institute (EMI), Centre of Biomedical Engineering of TTU (CBE of TTU) have all paid attention to studies of nonlinear processes.

In 1991 - 1998, 8 Annual Reports of the IOC at TTU, entitled "Nonlinear Dynamics: Theory and Applications" reflected results of one team in this context. In 1999, Centre for Nonlinear Studies (CENS) was founded at IOC in order to create synergism between all the teams. International Advisory Board (IAB) has been formed and fixed by the Council of IOC in order to link the activities of CENS directly to the International community.

This Report is the first Annual Report of CENS. It unites the above-mentioned series of IOC Reports with Reports of other research teams in this field.

In Section 2, the structure of CENS and its aims are briefly described. Section 3 forms the main part of this Report: the abstracts of all the studies are presented. Section 4 lists all the research grants related to these studies. Further, in Section 5 the overview of formal activities is given including the lists of published papers, conferences, theses, lecture courses, etc. The summary and the further prospects are described in Section 6.

# 2. Centre for Nonlinear Studies CENS; its structure and aims

CENS in the Institute of Cybernetics at Tallinn TU is created in order to concentrate the research efforts in nonlinear dynamics and related areas. Focussed aims of CENS are:

- to be at the frontier of science;
- to react to national interests;
- to participate in international research;
- to keep a good research atmosphere;
- to disseminate knowledge.

Research groups primarily in nonlinear dynamics are:

- dynamics of microstructured materials and solitons (P.I. - dr. A.Salupere);
- biomechanics and biophysics ( P.I. - dr. O.Kongas);
- general wave theory (P.I. - dr. A.Ravasoo).

Associated groups in related areas are:

- optical nonlinearity and photoelasticity (P.I. - prof. H.Aben, IOC);
- geometric approach to nonlinear problems (P.I. - prof. M.Rahula, Tartu University);
- wave interactions (P.I. - dr. T.Soomere, Estonian Marine Institute);
- biosignals interpretation (P.I. - prof. H.Hinrikus, TTU).

Based on the capacity of working groups, CENS is a cluster-type research unit with different links between them but research focussed on studies of complex systems. The Scientific Council of the IOC has nominated prof. J.Engelbrecht to act as a P.I. for all the CENS. In addition, the International Advisory Board (IAB) is formed by internationally well-known scientists.

The IAB is expected:

- to advise on general streamlines of research and (if needed) on local activities;
- to keep an eye on the quality of current projects;
- to support joint research;

The IAB members are:

Prof. Josef Ballmann (RWTH Aachen);
Prof. David G. Crighton (Cambridge University, DAMTP);
Prof. Bengt Lundberg (Uppsala University);
Prof. Gérard A. Maugin (University of Paris 6);
Prof. Valdur Saks (University of Grenoble);
Dr. Andras Szekeres (Budapest Technical University);
Prof. Dick van Campen (Eindhoven University of Technology);
Prof. Erik van Groesen (Twente University).

The list of hot problems as it stands in 1999 is the following:

(i) dynamics of microstructured materials and solitons:

- soliton dynamics in materials with complicated nonlinearities and higher order dispersion;
- virtual (short-living) solitons, general theory;
- physical interpretation of microstructure based on theory of fractals and dispersion laws;
- interaction of 2D solitons and the Hirota method;
- interaction of solitons over changing bottom;
- inverse problems of microstructured materials;
- hierarchical wave processes in microstructured materials.

(ii) biomechanics and biophysics:

- mechanical contraction of the heart muscle in dependence of inhomogeneities in "food" and oxygen supply;
- consistency of physical and mathematical modelling of intracellular energy fluxes;
- fractal analysis of the heart rate variability;
- fractal analysis of the blood-vessel systems and flow through fractal structures;
- hierarchical internal variables for modelling mechanical contraction of muscles.

(iii) general wave theory:

- inverse problems of nonlinear acoustodiagnostics;
- waves propagation and interaction in inhomogeneous materials;
- two-dimensional modelling of dynamical processes in inhomogeneous solids;
- thermodynamic consistency of numerical schemes for conservation laws;
- experimental testing of piano hammers;
- piano hammer-string interaction simulation.

(iv) optical nonlinearity:
- nonlinear optical phenomena on optical tomography of inhomogeneous birefringent media.

(v) geometric approach to nonlinear problems:
- catastrophes;
- gauge theories.

(vi) wave interactions and turbulence:
- general theory of kinetic equations;
- wave propagation and interactions in layered medium;
- modelling of wind waves;
- interactions of wind waves in strongly stratified flows;
- theory of rotationally anisotropic turbulence;
- generation of layered structure in the oceans;
- diffusion properties in rotationally anisotropic turbulence;
- generation of zonal flow by Rossby waves.

(vii) biosignals interpretation:
- nonlinear filtering and modelling algorithms;
- nonlinear modelling and classification of EEG signals under non-ionizing radiation;
- temporal and spatial variations of the ECG signals;
- interaction of waves in laser active media and selfmixing in lasers.

## 3. Main results

### 3.1 Institute of Cybernetics, Tallinn Technical University.

#### 3.1.1 Dynamics of microstructured materials and solitons.

**Soliton dynamics in the framework of microstructured solids.**
A Korteweg-de Vries type evolution equation is used for modeling the wave propagation in the microstructured media, i.e., in the media characterized by higher order dispersion and nonlinearity. The model equation has the quartic nonlinear term and both, the third order and the fifth order dispersive terms.
1. Normal and anomal dispersion domains are detected in the dispersion parameter plane and combined with solution types detected earlier. In the case of the KdV equation the dispersion is normal, i.e., the phase velocity is higher than the group velocity for any value of the dispersion parameter. However, for the studied evolution equation the character of the dispersion depends on the mutual relation of the third and the fifth order dispersion parameters. If the fifth order dispersion is dominant over that of the third order then the dispersion is anomal for any wavenumber ($k > 1$). In this domain a train of positive solitons forms from the initial harmonic wave. Vice versa, if the third order dispersion is dominant over that of the fifth order then the dispersion is anomal only for very high wavenumbers (corresponding to short wavelength harmonics) and then normal dispersion is dominating. In this case a train of negative solitons forms from the initial harmonic wave.
2. The specific influence of the quartic nonlinearity to the formation of soliton train is established. As a rule, soliton train starts to form in locations where the sine wave

breaks down in the nondispersive case. The quadratic nonlinearity in the KdV equation causes (in nondispersive case) one discontinuity in the sine wave. In the case of the quartic nonlinearity two additional discontinuities can be detected and the train of solitons forms namely near these two new discontinuities. The solution type depends now on the character of dispersion.

3. In the case of localized initial conditions, the influence of physical parameters on the character of propagation and interaction of solitary waves is elucidated. The research (A.Salupere, J.Engelbrecht, O.Ilison) is continued.

**Surface solitons**

The surface of a layer of fluid (water, for instance) in motion can show a variety of "patterns" formed by wave crests that diminish or increase in height, disappear, propagate, collide with other crests, etc. Even in the simplest situations (no wind, no current, etc), the free surface equations are well known but too complicated for a direct investigation of the wave patterns. So, an alternative approach should be considered, as we do, by describing the wave phenomena with soliton-type models. This is supported by the observation that "soliton" interactions seem to be rather robust phenomena in (the theory of) water waves.

The motivation of the following research has been motivated from practical questions from hydrodynamic laboratories where waves are generated to test ships. The so-called "inverse problem" of multi-directional waves concerns the question if from a (photographic) record of the wave pattern the interacting waves (wave heights, directions, etc) can be reconstructed. Since linear interactions do not disturb the interacting waves, nonlinear effects are to be taken into account.

The main results of our study are the following (P.Peterson):

- The inverse problem for wave crests is introduced. A solution strategy for two-waves interactions is given.
- Taking the model of water waves the KP (Kadomtsev-Petviashvili) equation, that describes (small but finite amplitude, long) surface waves travelling in shallow water (of constant depth) "mainly" in one direction, it is proved that the inverse problem has a unique solution.
- Actual solutions for the KP two-wave interactions are constructed, in particular for the cases with small interaction angle, moderate phase shifts, and/or symmetric interactions.
- Sensitivity of the inverse wave crest problem is investigated. As a result, the question of the practical applicability of the method is answered: the method of calculating the heights of the waves from a "photographic" record is applicable if the positions of the waves crests can be well defined from the wave pattern and the interaction angle is not too small.

### 3.1.2 Biomechanics and biophysics

**Mathematical modelling of intracellular energy fluxes**

The purpose of this research is to develop the quantitative methods - mathematical models - of compartmentalised energy fluxes of living cells, using the results of modern experimental research. This year, our main aim was to demonstrate by mathematical modeling, which of the intracellular metabolites may regulate (mediate the signal to) oxidative phosphorylation (OxPhos) under various enzymic conditions of the cell to balance the muscle cell's energy requirements. For this, we computed fractional response coefficients (FRCs) for the model of OxPhos and energy transfer for the rat heart that uses fatty acids as substrates. We observed that by inhibiting progressively creatine kinase (CK), the shift from CK to adenylate kinase (AK) catalysed energy transfer was accompanied by the shift from $P_i$- to AMP-mediated regulation of OxPhos. By inhibiting also AK reaction led to the drop of maximal respiration rate from 47 to 30 mmol $O_2$/(kgdw·min) accompanied by the shift from AMP- to ADP-mediated regulation. For $VO_2 = 80$ mmol $O_2$/(kgdw·min) and CK active, the raise in $P_i$ level was accompanied by the shift of regulation from $P_i$ to other metabolites. We conclude that the regulator metabolite for OxPhos may depend on the enzymic state of the cell, substrate, and muscle type (O.Kongas, M.Vendelin, J.Engelbrecht, V.Saks (KBFI, Estonia)).

**Mechanical contraction of the heart muscle.**
The long-term goal of the project is to estimate the local oxygen consumption of the cardiac muscle from the deformation measurements. This year, we reproduced the experimentally measured linear dependence of the cardiac muscle oxygen consumption on stress-strain area using the Huxley-type model. By selecting particular cross-bridge cycling rate constants and modifying the cross-bridge activation model, we replicated the dependence between oxygen consumption and stress-strain area together with other important mechanical properties of cardiac muscle such as developed stress dependence on the sarcomere length and force-velocity relationship. The model predicts that (1) the amount of the "passenger" cross-bridges, i.e. cross-bridges that detach without hydrolyzing ATP molecule, is relatively small and (2) ATP consumption rate profile within a beat and the amount of the "passenger" cross-bridges depend on the contraction protocol (M.Vendelin, J.Engelbrecht, T.Arts (Maastricht University, the Netherlands), P.Bovendeerd and D.van Campen (Eindhoven University of Technology, the Netherlands)).

**Fractality**
The passive scalar convection by chaotic two-dimensional incompressible flow has been studied. Analytically solvable equations were suggested to describe the evolution of the probability density functions of tracer gradients and power spectra. The parameters of the model were expressed explicitly via the porrelation functions of the velocity field. The multifractal spectrum $f(\alpha)$ of the scalar dissipation field has been calculated. The strict multifractality holds only for small values of $\alpha$. Stationary and exponentially decaying power spectra of the scalar have been obtained. The results are in a good agreement with experimental data. In order to model the potential damage of oceanic or atmospheric pollution, new multifractal scalar fields have been introduced.

The transport of passive scalar by random velocity fields is related to the problem of the statistical properties of streamlines. In two-dimensional geometry, the streamlines are given by the isolines of random surfaces. In order to model the isolines of random Brownian and fractional Brownian surfaces, new percolation-type lattice model has been suggested. The Monte-Carlo simulations based on this model are extremely fast. Thus, the computer time $t$ needed to generate a single isoline of size $a$ scales as $t \propto a^d$; the fractal dimension of separate isolines has been found to be $d = 1.28074 \pm 0.000005$. Note that the conventional methods lead to $t \propto a^3$.

The research results of past few years have been used to prepare two papers devoted to the tree-like fractal biological networks, and particularly to the blood-vessel system (J.Kalda).

**Heart rate variability**
The applicability of the methods of nonlinear dynamics to the analysis of the heart rate variability (HRV) has been studied in cooperation with Tallinn Diagnostic Center and Nõmme Hospital. The traditional phase-space-based analysis is based on the idea that severe and chronic illnesses decrease the number of freedoms in the dynamical system responsible for the heart rhythm generation. This approach has been questioned; it has been argued that for most patients, the apparent structures in phase-space are due to the heart rate modulation by respiration.

A new Zipf's-law-based multiscaling behavior of the HRV has been discovered. A threshold parameter $\delta_0$ has been used to define the low-variability periods. These periods were arranged according to their durations $T$. In such a way, each low-variability period obtained a rank $r$. For a certain range of the values of $\delta_0$, the scaling behavior $T \propto r^{-\gamma}$ has been observed. For healthy patients, the scaling exponent $\gamma$ revealed a significant dependence on the parameter $\delta_0$. (J.Kalda and M.Säkki).

**Hierarchical internal variables**
The formalism of internal variables widely used in continuum mechanics, is proposed for describing the processes of deformation in cardiac muscles. Due to complicated hierarchical microstructure of soft tissues where macroscopic stress states depend upon the sliding of molecules and ion concentrations, the internal variables form a certain hierarchy. This seems to be a general concept for complicated, materials with different microscopic processes influencing the macroscopic behaviour. The concept of hierarchical internal variables is formulated in terms of continuum mechanics. The analysis is carried out on the example of Huxley-type models for cardiac muscles (J.Engelbrecht, M.Vendelin, G.A.Maugin).

### 3.1.3 General wave theory

**Acoustodiagnostics of inhomogeneous and prestressed solids.**
The applicability of nonlinear acoustodiagnostics method proposed in 1998 for nondestructive testing of dispersive materials, such as physically inhomogeneous material, homogeneous material undergoing inhomogeneous predeformation, etc., is studied in detail. The method is based on the simultaneous propagation of two longitudinal waves and interaction data analyses.

The governing equations of the method for acoustodiagnostics of nonlinear elastic material properties are ordered with the view to utilize the information in sine-wave harmonic amplitudes and phase shifts evolution data for material characterization. It is shown that for certain values of initial frequencies of waves excited simultaneously in homogeneous nonlinear elastic material, the analysis may be simplified essentially. Namely, it is possible to describe the harmonic and phase shift evolution on the material boundaries by functions that contain only one Heaviside function in the wave propagation and the wave interaction intervals, respectively. This enables to derive the explicit analytical expressions for all amplitudes and phase shifts of harmonics in both intervals. The result is that the material characterization algorithm simplifies essentially. It may be formulated on the basis of harmonics amplitudes and phase shifts measurement data in both intervals and on both material boundaries making use of the fact that the amplitudes and phase shifts have the different sensitivity to variation of the values of material properties.

The mutual nonlinear interaction features of three first harmonics of sine-wave in various regions of nonlinear elastic material are clarified. The result may be used, for example, in nondestructive testing of rods on the basis of wave propagation measurement data recorded not only at the ends of the rod but in the intermediate sections, also.

The influence of inhomogeneous plane strain on nonlinear effects of longitudinal wave propagation, reflection and interaction in nonlinear elastic material is investigated. The sensitivity of nonlinear distortion of the wave profile to the predeformed state parameters is clarified. The essential difference in evolution of nonlinear effects in intervals of wave propagation and nonlinear interaction is noticed. It is important for nondestructive testing that on the material boundaries there is the different dependence of wave profile distortion on the sign and the value of the predeformed state parameters in both intervals. The inhomogeneity of the predeformed state can be easily determined by the difference in recorded wave profiles on the boundaries. These results enable to propose effective algorithms for nondestructive characterization of inhomogeneously predeformed materials. The elaboration of these algorithms and the expansion of these ideas to other types of inhomogeneous materials is the scope of further research A.Ravasoo, A.Braunbrück).

**Thermomechanics of complex systems**

A novel approach to the modelling of thermoelastic wave propagation is developed based on the thermodynamics of discrete systems. The novelty consists in the representation of integral balance laws for thermoelasticity in terms of contact quantities that describe non-equilibrium state of discrete elements, which represent a continuous medium. In the required extension of the thermodynamics of discrete systems to the thermoelastic case, in addition to contact temperature that governs heat exchange we should define a contact stress tensor since the state space includes the deformation. It should be noted that the values of contact quantities differ from the values of usual bulk parameters. For interacting elements, the values of bulk and contact quantities of adjacent elements are additionally connected by the thermodynamic consistency conditions. The thermodynamic consistency conditions are the consequences of the first law of thermodynamics in the case of interacting discrete systems. Such a generalization leads to a natural finite-volume scheme for the numerical simulation of two-dimensional thermoelastic wave propagation in inhomogeneous media. The satisfaction of thermodynamic consistency conditions between adjacent discrete elements provides the physical consistency of the algorithm (A.Berezovski).

**Study of acoustical spectra of piano strings**

The piano hammer parameters measuring device was developed and built in the Institute of Cybernetics at TTU. This equipment gives a possibility to investigate the dynamical force-compression characteristics of piano hammer, and, using the hereditary (hysteretic) hammer model, to find the hammer parameters by numerical simulation of the dynamical experiments.

This device consists of three main parts. The first mechanical part gives the needed velocity of interaction of the hammer with the string. The second and the third parts are composed by a piezoelectric wide-band force sensor, and an infrared optical sensor for registration of hammer deformation. The analogue force-time and deformation-time signals from these two sensors are converted into two sets of data by a digital signal processor. Using these data, the force-compression characteristics of the hammer may be obtained for any hammer velocity. The piano hammer parameters measuring device in conjunction with the hysteretic model of the piano hammer is a powerful instrument as for matching the piano hammers, as well as for their manufacturing.

In fact, the numerical simulation using the hysteretic hammer model may significantly simplify the process of the piano hammers manufacturing. Actually, there are many parameters (e.g. the instantaneous hammer stiffness, the stiffness nonlinearity exponent, mass, radius of curvature, the hereditary parameters, and so on) describing the properties of a hammer. The conditions of piano hammers manufacturing vary also on a large scale. By using of this measuring device it is possible to find the dependencies of the hammer parameters on the technological conditions during manufacturing. Therefore the knowledge of these dependencies gives a good practical hint to choose a better technological process for the hammers manufacturing. In this sense, the hysteretic hammer model is an irreplaceable model.

The results of dynamical measurements of piano hammers are very important. It is very interesting to prove the hysteretic hammer model in practice, also. The combination of the experimental piano hammer testing and the hysteretic hammer model together is the good practical way to improve the hammer quality and thus, the piano voicing. By numerical simulation of the hysteretic model it is possible to calculate the hammer parameters that provide the needed spectrum of the piano string vibrations.

The analysis of the hammer-string interaction shows that the nonlinear hysteretic model of piano hammer provides predictions about the vibration spectra of struck strings for real pianos that come closer to measured data, than predicts the nonhysteretic model. In addition to the correct spectra, the hysteretic model gives more suitable values of the hammer compression. A piano hammer parameters measuring device gives the possibility to find the dependencies of the hammer parameters on the key number. It seems that, for the one set of the good hammers such an explicit dependence exists. The numerical simulation of the known data shows, that hereditary amplitude increases and relaxation time decreases definitely with a key number. Probably, the value of $p$ increases with a key number also.

The hammer stiffness is a constant value in hysteretic model. This parameter depends on the hammer size, wear, manufacturers. For one certain set of piano hammers it is possible to find experimentally the value of the instantaneous hammer stiffness. The knowledge of the hammer parameter dependencies on the key number permits to produce the hammers with the needed features (A.Stulov).

**General problems**

A modification of the wave-propagation algorithm is used as a tool for determining contact quantities in a finite-volume scheme for the numerical simulation of two-dimensional thermoelastic wave propagation in inhomogeneous media. The wave-propagation algorithm provides the second-order resolution and the stability of the numerical scheme up to the Courant number 1. It is well known, however, that the lack of monotonicity for higher order numerical methods is reflected by spurious oscillations in the vicinity of jump discontinuities. The usual way to supress these oscillations is the introduction of certain limiter functions. However, the use of non-linear wave limiters may destroy the superposition principle (at least for linear thermoelasticity). In addition, the limiters may lead to problems of the consistency between adjacent discrete elemets in numerical simulation. In particular, the satisfaction of the thermodynamic consistency conditions which follow from the thermodynamics of discrete systems cannot be controlled immediately. Therefore, a composite scheme is proposed where the Godunov step is applied after each three second-order Lax-Wendroff steps. This combination keeps the advantage of the wave-propagation algorithm and allows us to avoid the application of limiter functions that are not consistent with differential equations. Elimination of source terms is made following the method of balancing source terms after independent solution of the heat conduction equation. This implies a redefinition of the bulk velocities to account of the temperature derivative. the thermodynamic consistency conditions are fullfilled in the procedure. Results of computation for certain test problems show the efficiency and physical consistency of the algorithm (A.Berezovski).

### 3.1.4 Optical nonlinearity and photoelasticity

The structure of the isochromatic fringe pattern of a two-dimensional photoelastic model has been investigated using the phase diagram method of the theory of dynamic systems. The level curves of the light intensity are considered as the phase paths of a Hamiltonian dynamic system. Singularities of this system are investigated. There may exist singular curves and singular points. Singular curves are isochromatic fringes. Singular points can be either centres, saddle points, or one has a degenerate case. These results can be used by numerical treatment of the isochromatic fringe patterns and they help to understand the nonlinear optical phenomena that are observed in fringe patterns of three-dimensional photoelastic models.

A method for complete stress measurement in step-index optical fibres and fibre preforms has been developed. Axial stress distribution is determined using integrated photoelasticity. In every layer of the fibre stress is approximated by a polynomial; at the borders of the layers the stress is discountinuous. For the calculation of the circumferential stress the classical sum rule is generalized for the case of an inhomogeneous cylinder.

Several new measurement techniques were elaborated for Verrerie Cristallerie d'Arques for residual stress measurement in tempered and annealed glassware. Research carried on by H.Aben, L.Ainola, J.Anton.

## 3.2 Tartu University

### 3.2.1 Geometric approach to nonlinear problems

It has been shown that an exponential law whick holds in a space of mappings has a universal property. Namely, any motion along an orbit of a Lie group can be described in an invariant way by a jet structure of mappings. This result allows to study the symmetries of differential equations with the help of the Cartan method which means a cetrain breakthrough in the studies of differential geometry and differential equations.

A generalization of exterior calculus with exterior differential $d$ satisfying $d^N = 0,\ N > 2$ is constructed and studied. The construction of generalization is based on the notion of graded q-differential algebra and non-communtative first order differential calculus. A peculiar property of the above mentioned generalization is the necessity to introduce the second (and higher) order differentials. A bimodule structure of these differentials is studied and it is shown that in dimension one the structure of bimodule of higher order differentials is consistent with the structure of bimodule of first order differentials only in the case of braided (quantum) line. A connection and curvature of a generalized exterior calculus with $d^3 = 0$ on reduced quantum plane is studied. Research carried on by M.Rahula and V.Abramov.

## 3.3 Estonian Marine Institute

### 3.3.1 Wave interactions

**Rossby waves and zonal flow**

The Rossby wave is an example of low frequency oscillations found in rotating systems. A remarkable property of this type of motions in nature (called synoptic motions) is energy transfer to larger scales. The process is accompanied by supporting an anisotropic flow directed along parallels and called zonal flow.

Analysis of the results of an experimental study of Rossby wave interactions in the "Coriolis" rotating tank (ø13m, Grenoble University) confirmed that travelling waves excited a noticeable zonal transport in the form of weak jets after ca 10 wave periods. In some cases, wave-wave interactions lead to water displacements by a distance comparable to the wave amplitude. The experiments confirm the prediction of the weakly nonlinear (kinetic) theory that propagation of Rossby waves may result in a considerable zonal transport of water masses. The detected striped transport arose in some cases surprisingly fast and had different structure in the subsequent experiments with identical parameters. This feature suggests that it has indeed random nature and emerges as a result of Rossby-wave interactions (T.Soomere, T.Koppel).

**Joint evolution of classical and generalised spectra**

Detailed study of the above-mentioned peculiarity of the synoptic motions has revealed the existence of generalised anisotropic, thermodynamically equilibrated spectra of such motions. Owing to the "balance" of the nonlinearity. Rossby-wave systems evolve towards a particular equilibrated state, consisting of a superposition of a zonal flow and a spectrally isotropic wave system.

Although the parts of motions described by different types of spectra can be distinguished mathematically rather than physically, their evolution scenarios are basically different. Earlier results obtained for systems of Rossby waves have been generalised to the case of arbitrary wave systems allowing three-wave interactions. Spectral evolution of such wave systems was shown to be irreversible. The existence and stability of thermodynamically equilibrated spectra were proved. A deeply interesting feature of the presence of a singular spectrum is that it accelerates evolution of the smooth part towards thermodynamical equilibrium. Moreover, its presence additionally suppresses amplitude of small disturbances of the classical equilibrium spectra. In other words, wave systems with generalised spectra serve as a catalyst of the evolution of the whole field towards equilibrium even if they do not change themselves.

**Evolution of a cluster of closely packed intensive synoptic vortices**

The evolution of synoptic vortices of one sign (cyclonic or anticyclonic) in 2D anisotropic turbulence (homogeneous medium on a b-plane) was studied experimentally in the "Coriolis" rotating tank (ø13m).

In several experiments with anticyclones, an unexpected interaction of vortex dipoles has been observed. Namely, concentrated anticyclons penetrated into cyclons and formed practically axisymmetric mostly cyclonic structures with a strong anticyclonic kernel and an intermediate zone with nearly zero vorticity. Owing to the essential difference in the vortices' intensities, the process can rather be explained as axisymmetrization of dipoles than fortuitous combination of flow structures. The process might explain why some hurricanes have an anticyclonic kernel. A specific feature of the observed process is that it occured in a vertically homogeneous medium while anologous phenomena in nature (e.g. Jupiter's Red Spot, which, however, is a giant anticyclone with a cyclonic kernel) have been explained, until now, basically through stratification effects.

In the experiments with cyclones, the anomalous pairing of small cyclons with comparable intensity has been observed. They formed a vortex pair which persisted during many of its components turnover times until it drifted out of the measurement area. The distance between the pair's components was less than twice the components diameter. The pairs slowly rotated counterclockwise and generally behaved as a cyclon comparable with the pairs' size. This is an infrequent case of a "double monopole" or a dipole consisting of two elements of the same sign. Detailed investigation of both the anomalous processes is in progress.

**Theory of turbulence**

In the form of a monography, a new systematic and metatheoretic concept of hierarchic stochastic systems has been formulated. Such systems have been described in terms of a hierarchy of theories based on different coding systems and satisying certain invariance conditions. In particular, hydrodynamical medium has been considered as an example of such a system. The monography has mainly been focused on turbulent motions. Principles of rotationally anisotropic description of turbulent fields have been formulated. In different from classical understanding of turbulence, rotational displacements form an additional degree of freedom of the motions in question. A version of moment equation is introduced in order to describe effects created by rotational anisotropy. A selection of applications has been described in detail.

### 3.4 Centre of Biomedical Engineering, Tallinn Technical University

#### 3.4.1 Biosignals interpretations

The different nonlinear L-filtering, Stack filtering and Linear-WOS (Weighted Order Statistic) Hybrid filtering have been applied for the analysis ECG and EEG signals. The general ideas behind the mentioned filtering algorithms, most useful subclasses of the filters and their application in physiological signal processing were analysed. Adaptive WOS filters to EEG analysis and modelling will be applied in near future. A graphical front end to MatLab environment was developed, aimed for visual analysis and processing of polygraphic recordings of physiological signals in European Data Format (EDF). The developed system enables to apply user-defined signal processing algorithms to graphically selected segments of the signals contained in EDF recordings. It is shown that depression of alpha-waves in human EEG takes place after modulated low-level microwave exposure. The simple coherent optical method based on nonlinear effects in laser active media was developed for pulse wave shape and velocity measurement. It is proved experimentally that the coherence length of light in the case of selfmixing is much longer than for traditional interferometry.

## 4. Grants

1. BMBF (Germany) grant 03 N 9005 2. Nonlinear dynamics of heterogeneous solids with microstructure, Partner - RWTH Aachen. Supervisors: J.Ballmann (Aachen), J.Engelbrecht (Tallinn).
2. ETF grant N 2631. Solitons in microstructured materials (A.Salupere).
3. ETF grant N 3203. Two-dimensional wave processes in continua with microstructure (A.Stulov).
4. ETF grant N 3204. Mathematical modelling of intracellular energy fluxes (J.Engelbrecht).
5. ETF grant N 3739. Self-consistent fractal modelling of the oxygen supply and contraction in myocardium (J.Kalda).
6. ETF grant N 3740. Analysis of the heart rate variability using the methods of nonlinear dynamics (O.Kongas).
7. EIF grant. Piano hammer parameters measuring device (A.Stulov).
8. ETF grant N 3595. Photoelastic methods for nondestructive testing of stress field (H.Aben).
9. ETF grant N 3411. Electromagnetic sensitivity of biological systems (H.Hinrikus).
10. ETF grant N 1891. Selfmixing in laser and its application in medical diagnostics (K.Meigas).

# 5. Publicity of Results

## 5.1 Research Reports

| | | |
|---|---|---|
| 1. | Mech 199/99 | J.Kalda, O.Kongas, M.Vendelin et.al. Modelling of cardiac phenomena. |
| 2. | Mech 200/99 | J.Engelbrecht. A nonlinear story of nonlinear waves. |
| 3. | Mech 201/99 | O.Kongas, M.Vendelin, J.Engelbrecht, V.Saks. Studies in metabolic control analysis. |
| 4. | Mech 202/99 | A.Salupere, J.Engelbrecht, O.Ilison, G.A.Maugin. Numerical Analysis of Solitonic Structures in Systems with Higher-Order Dispersion and Nonlinearity. |
| 5. | Mech 203/99 | J.Engelbrecht, M.Vendelin, G.A.Maugin. Hierarchical Internal Variables Reflecting Microstructural Properties: Application to Cardiac Muscle Contraction. |
| 6. | Mech 204/99 | A.Salupere, J.Engelbrecht, G.A.Maugin. Solitonic structures in KdV - based higher-order systems. |
| 7. | Mech 205/99 | A.Berezovski, G.A.Maugin. Simulation of thermoelastic wave propagation by means of a composite wave-propagation algorithm. |
| 8. | Mech 206/99 | A.Berezovski, G.A.Maugin. Application of wave-propagation algorithm to 2D thermoelastic wave propagation in inhomogeneous media. |
| 9. | Mech 207/99 | H.Aben, L.Ainola. On the theory of photoelastic fringe patterns. |
| 10. | Mech 208/99 | A.Ravasoo, B.Lundberg. Nonlinear interaction of longitudinal waves in an inhomogeneously predeformed elastic medium. |
| 11. | Mech 209/99 | A.Ravasoo. Nonlinear longitudinal waves interaction for inhomogeneously predeformed medium characterization. |
| 12. | Mech 210/99 | A.Braunbrück, A.Ravasoo. Symbolic software application for modelling of wave interaction process. |
| 13. | Mech 211/99 | T.Sillat. Wave propagation in dissipative microstructured materials. |
| 14. | Mech 212/99 | A.Berezovski, J.Engelbrecht, G.A.Maugin. Thermoelastic wave propagation in inhomogeneous media. |
| 15. | Mech 213/99 | A.Ravasoo, J.Janno. Nondestructive characterization of materials with variable properties. |
| 16. | Mech 214/99 | M.Vendelin, P.H.M.Bovendeerd, T.Arts, J.Engelbrecht, D.H.van Campen. Cardiac mechanoenergetics replicated by cross-bridge model. |
| 17. | Mech 215/99 | A.Stulov. Piano hammer characteristics: theory and experiment. |
| 18. | Mech 216/99 | P.Peterson, E.van Groesen. Direct and inverse problem for wave crests modelled by soliton interactions. |

### 5.2 Publications

**Books**

1. Ü.Lepik, J.Engelbrecht. The book of Chaos, Estonian Academy Publishers, Tallinn, 1999 (in Estonian).
2. J.Heinloo. The mechanics of turbulence. Estonian Acad. Publ., Tallinn, 1999, 270pp.

**Papers (refereed)**

1. A.Berezovski. Modeling of two-dimensional elastic wave propagation with continuous cellular automata. Proc. Estonian Acad. Sci. Eng., 1999, 5, 2, 112-121.
2. A.Berezovski, G.A.Maugin. Material formulation of finite-strain thermoelasticity and applications. J. Thermal Stresses, 1999, 22, 421-449.
3. A.Braunbrück, A.Ravasoo. Symbolic software application for modelling of wave interaction process. In: Proc. 12th Nordic Seminar on Comput. Mech. 22-23 Oct. 1999, Helsinki, HUT, 89 - 92.
4. J.Engelbrecht, R. von Hertzen, O.Kongas. Driven oscillators for modelling cardiac phenomena. In: F.C.Moon (ed), Proc. IUTAM Symp. New Applications of Nonlinear and Chaotic Dynamics in Mechanics. Kluwer, Dordrecht, 1999, 333-342.
5. J.Engelbrecht, A.Mägi, A.Stulov. Grand piano manufacturing in Estonia: The problem of piano scaling. Proc. Estonian Acad. Sci. Eng., 1999, 5, 2, 155-167.
6. J.Engelbrecht, A.Ravasoo, A.Salupere. Stress and solitary waves in solids. In: L.Fryba and J.Naprstek (eds.), Structural Dynamics, I, Balkema Publ., Rotterdam, 1999, 23-30.
7. J.Engelbrecht, F.Pastrone, P.Cermelli. Wave hierarchy in microstructured solids. In: G.A.Maugin (ed.), Geometry, Mechanics, Microstructure. Hermann Publ., Paris, 1999.
8. J.Engelbrecht, M.Vendelin, O.Kongas, R. von Hertzen. Cardiac dynamics involving arrhythmias and continuum mechanics. In: B.T.Maruszewski, W.Muschik, and A.Radowicz, (eds.), Trends in Continuum Physics TRECOP 98, World Scientific, Singapore et al., 1999, 113-118.
9. J.Engelbrecht, A.Salupere, P.Peterson. Nonlinear wave motion: complexity and simplicity revisited. In: E.Lavendelis (ed.), Proc. IUTAM Symp. Synthesis of Nonlinear Dynamical Systems. Kluwer, Dordrecht, 2000, 25-36.
10. J.Kalda. Transport processes in fractal biological networks. Proc. Est. Acad. Sci. Eng., 1999, 4, 270-280.
11. J.Kalda, M.Säkki, M.Vainu, M.Laan. The methods of nonlinear dynamics in the analysis of heart rate variability for children. Med. Biol. Engineering & Computing, 1999, 37, Suppl. 1, 69-72.

12. O.Kongas. A global map of local bifurcations. In: E.Lavendelis (ed.), Proc. IUTAM Symp. Synthesis of Nonlinear Dynamical Systems. Kluwer, Dordrecht, 1999, 179-188.

13. O.Kongas, M.Vendelin, V.Saks. Modeling of intracellular compartmentalized energy and metabolic fluxes in the heart. Med. Biol. Engineering & Computing, 1999, 37, Suppl. 1, 27-32.

14. O.Kongas, R.von Hertzen, J.Engelbrecht. Bifurcation structure of a periodically driven nerve pulse equation modelling cardiac contraction. Chaos, Solitons & Fractals, 1999, 10, 1, 119-136.

15. O.Kongas, M.Vendelin. Mathematical modelling of cardiac phenomena: arrhythmias, cell energetics, and contraction. Proc. Estonian Acad. Sci. Phys. Math., 1999, 48, 3/4, 278-287.

16. M.Kutser. Contemporary trends in Estonian mechanics. Proc. Estonian Acad. Sci. Eng., 1999, 5, 2, 168-178.

17. A.Ravasoo, A.Braunbrück. Nonlinear interaction of longitudinal waves in elastic material. Proc. Estonian Acad. Sci. Phys. Math. 1999, 48, 3/4, 252-264.

18. A.Salupere, J.Engelbrecht, O.Ilison, G.A.Maugin. Numerical analysis of solitonic structures in systems with higher-order dispersion and nonlinearity. In: W.Wunderlich (ed.). Proc. Eur. Conf. Comp. Mechanics 1999. CD-ROM, TU München, 1999, 1-17.

19. A.Stulov. Method of piano hammer parameters determination. In: Collected Papers of 137th meeting of the Acoustical Society of America and the 2nd Convention of the European Acoustics Assotiation: Forum Acusticum, 1999. CD-ROM, ASA/EAA, Berlin, 1999, 1-6.

20. M.Vendelin, P.Bovendeerd, T.Arts, J.Engelbrecht, D.H. van Campen. Linear dependence of myocardium oxygen consumption on stress-strain area predicted by cross-bridge model. Med. Biol. Engineering & Computing, 1999, 37, Suppl 1, 63-66.

21. A.Ravasoo. Nonlinear longitudinal waves in inhomogeneously predeformed elastic media. J.Acoust. Soc. Amer. 1999, 106, 6, 3143-3149.

22. V.Abramov, R.Kerner. On certain realizations of the q-deformed exterior differential calculus. Reports on Math. Phys., 43, 1/2,179-194, 1999.

23. V.Abramov, M.Rahula. Differential geometry in Tartu in 20th century. In: Ülo Lumiste, mathematician, Tartu, 127-139, 1999.

24. M.Rahula. Exponential law in the Lie-Cartan calculus. Rendiconti del Seminaro de Messina. Ser- II, 1999, XIV, 3, 231-250.

25. T.Soomere, T.Koppel. Linear Rossby waves and zonal transport in rotating platform. Proc. Estonian Acad. Sci. Engineering, 5, 1, 22-40, 1999.

26. T.Soomere, T.Koppel. Generation of zonal transport by weakly nonlinear Rossby waves, in: Proc. XXVIII Congress of the International Asso-ciation for Hydraulic Research, Graz, Austria, 1999, 8 pp. (CD-ROM), 1999.

27. T.Soomere. Joint evolution of generalized and classical spectra in the kinetic theory, Proc. Estonian Acad. Sci. Phys. Math., 48, 3/4, 230-238, 1999.

28. H.Aben, L.Ainola, and J.Anton. Integrated photoelasticity as a tool for quality control in glass industry. In: GESA Symposium "Anspruch und Tendenzen in der experimentellen Strukturmechanik". Warnemünde, 1999, 233-238.

29. H.Aben, L.Ainola, and J.Anton. Sum rules for photoelastic residual stress measurement in axisymmetric glass articles. In: Proc. International Conference on Advanced Technology in Experimental Mechanics (ATEM'99). Ube, Japan, 1999, 2, 629-634.

30. L.Ainola and H.Aben. Duality in optical theory of twisted birefringent media. J. Opt. Soc. Amer. A, 1999, 16, 10, 2545-2549.

31. L.Ainola, H.Aben, and J.Anton. Half-fringe phase-stepping with separation of the principal stress directions. Proc. Estonian Acad. Sci. Eng., 1999, 5, 3, 198-211.

32. T.Lipping. Nonlinear Digital Filtering of Physiological Signals. Med. Biol. Eng. & Comp., 1999, vol.37, supp.1, 73-76.

33. J.Lass, V.Tuulik, and H.Hinrikus. Modulated Microwave Effects on EEG Alpha-Waves. Ibid., 105-108.

34. K.Meigas, R.Kattai, and J.Lass. Pulse Parameters Registration Using Self-Mixing in a Diode Laser as Optical Coherent Method. Ibid., 93-96.

35. J.Lass, H.Hinrikus, K.Meigas, and J.Kaik. Comparison of Different Heart Rate Reconstruction Algorithms for Rate Adaptive Cardiac Pacing. Med. Biol. Eng & Comp., 1999, vol. 37, supp. 2, Part II, 1282-1283.

36. K.Meigas, H.Hinrikus, R.Kattai, and J.Lass. Optical Method for Pulse Wave Velocity and Pulse Profile Registration. Ibid., 924-925.

37. T.Lipping and A.Anier. User Interface for Analysis of Polygraphic Recordings of Physiological Signals in European Data Format. Proc. Estonian Acad. Sci. Eng., 1999, 5/4, 304-311.

38. J.Lass, K.Meigas, H.Hinrikus, and J.Kaik. Noninvasive Multiparameter Registration System for Evaluating Physiological Condition. Ibid., 281-292.

39. K.Meigas, H.Hinrikus, R.Kattai, and J.Lass. Coherent Photodetection for Pulse Profile Registration. In: Proc. Coherence Domain Optical Methods in Biomedical Science and Clinical Applications III, SPIE, 27-29 Jan., 1999, San Jose, 195-202.

### Abstracts

1. A.Ravasoo. Nonlinear longitudinal waves interaction for inhomogeneously predeformed medium characterization. Int. Symp. Nonlinear Acoustics. Symposium Programm & Book of Abstracts. 1 - 4 Sept. 1999, Göttingen, Germany, p.52.

2. J.Engelbrecht, A.Salupere. Solitonic structures in KdV-based higher-order systems. The 4th Int. Congress on Industrial and Applied Mathematics. Book of Abstracts, 5 - 9 July, 1999, Edinburgh, Scotland, p.25.

3. A.Szekeres, J.Engelbrecht. Coupling of generalized heat and moisture transfer. Proc. of the ASME Mechanics and Materials Conf., June 27 - 30, 1999, Blacksburg, p.231.

4. A.Salupere, J.Engelbrecht, O.Ilison, G.A.Maugin. Analysis of Solitonic Structures in Systems with Higher-Order Dispersion and Nonlinearity. In Abstracts of the European Conference on Computational Mechanics - ECCM '99. August 31 - September 3, 1999, Munich, Germany, 274 - 275.

5. A. Berezovski and G.A. Maugin, Application of Wave-Propagation Algorithm to 2D Thermoelastic Wave Propagation in Inhomogeneous Media, Book of Abstracts of the International Conference "Godunov Methods: Theory and Applications", Oxford, UK, October 18-22, 1999.

6. K.Meigas, H.Hinrikus, R.Kattai, and J.Lass. Laser-Doppler Method for Suspension Flow Detection Ucing Optical Self-Mixing in Diode Laser. Physica Medica. Vol. XV, N 3, July - September 1999, p.190.

7. H.Hinrikus, J.Lass, K.Meigas, J.Riipulk, and V.Tuulik. Non-Ionising Radiation Effects on Brain Electrical Activity. Physica Medica. Vol. XV, N 3, July - September 1999, p.214.

8. H.Hinrikus, K.Meigas, J.Kaik, and J.Lass. Adaptive Algorithms for Heart rate Reconstruction. IEEE EMBS & EMBS Annual Meeting. Atlanta, Octoober 13-16, 1999, CD Proceedings, p.320.

**Submitted papers**

1. A.Berezovski, G.A.Maugin. Application of wave-propagation algorithm to 2D thermoelastic wave propagation in inhomogeneous media. Proc. Int. Conf. Godunov Methods: Theory and Applications. Oxford (to appear in 2000).

2. A.Berezovski and G.A.Maugin, Thermoelasticity of Inhomogeneous Solids and Finite-Volume Computations, K.Wilmanski Festschrift (accepted).

3. A.Berezovski and G.A.Maugin, Simulation of Thermoelastic Wave Propagation by Means of a Composite Wave-Propagation Algorithm, J. Comp. Physics (submitted).

4. J.Engelbrecht, M.Vendelin, G.A.Maugin. Hierarchical internal variables reflecting microstructural properties: application to cardiac muscle contraction. J. Non-Equil. Thermodyn. (under revision).

5. J.Kalda. Fractality of the biological tree-like structures. Discrete Dynamics in Nature and Society (accepted).

6. J.Kalda. Simple model of intermittent passive scalar turbulence. Phys. Rev. Lett. (accepted, scheduled for 84(2), 2000).

7. P.Peterson, E. van Groesen. Direct and inverse problem for wave crests modelled by soliton interactions. Physica D (revised and submitted).

8. P.Peterson, E. van Groesen. Sensitivity of the inverse problem. Wave Motion (submitted).

9. A.Ravasoo. Nonlinear longitudinal waves interaction for inhomogeneously predeformed medium characterization. Proc. Int. Symp. Nonlinear Acoustics. Göttingen, Germany (accepted).

10. A.Ravasoo, B.Lundberg. Nonlinear interaction of longitudinal waves in an inhomogeneously predeformed elastic medium. Wave Motion (submitted).

11. V.Saks, O.Kongas, M.Vendelin, L.Kay. Role of the creatine/phosphocreatine system in the regulation of mitochondrial respiration. Acta Physiol. Scand. (accepted).

12. V.Saks, M.Vendelin, O.Kongas, L.Kay. The creatine-phosphocreatine path way in the intracellular networks of energy transfer and signal transduction in muscle cells. In: R.Paoletti (ed.), Creatine: From Basic Science to Clinical Applications. Kluwer, Dordrecht (in press).

13. A.Salupere, J.Engelbrecht, G.A.Maugin. Solitonic structures in KdV - based higher-order systems. Wave Motion (submitted).

14. M.Vendelin, P.Bovendeerd, T.Arts, J.Engelbrecht, D.H. van Campen. Cardiac mechanoenergetics replicated by cross-bridge model. Annals of Biomed. Eng. (under revision).

15. M.Vendelin, O.Kongas, V.Saks. Regulation of mitochondrial respiration in heart cells analyzed by reaction - diffusion model of compartmentalized energy transfer. Amer. J. Phys. (accepted).

16. V.Abramov, R.Kerner. Exterior differentials of higher order and their covariant generalization. Preprint GCR - 98/02/03 (Universite Paris VI), Journal of Mathematical Physics (submitted).

17. M.Rahula. Loi exponentsielle, symetries des equations differentielles et repere de Cartan, C.R.Ac.Sci., Paris (submitted).

18. H.Aben and L.Ainola. Isochromatic fringes in photoelasticity. J.Opt. Soc. Amer. A, 2000, 17, 4.

19. L.Ainola and H.Aben. Hubrid mechanics for axisymmetric thermoelasticity problems. J. Thermal Stress.

20. H.Aben, L.Ainola, and J.Anton. Integrated photoelasticity for nondestructive residual stress measurement in glass. Accepted for presentation at the International Conference on Trends in Optical Nondestructive Testing, Lugano, May 3-5, 2000.

### 5.3 Conferences

1. J.Engelbrecht, A.Salupere. Solitonic structures in KdV-based higher-order systems. The Fourth International Congress on Industrial and Applied Mathematics - ICIAM 99. July 5 - 9, 1999, Edinburgh, Scotland, UK (invited lecture).

2. J.Engelbrecht; A.Ravasoo, A.Salupere. Stress and solitary waves in solids. Fourth Conference of the European Association for Structural Dynamics - EURODYN'99, Prague, Czech Republic, 7 - 10 June 1999 (invited key lecture).

3. A.Salupere, J.Engelbrecht, O.Ilison, G.A.Maugin. Numerical Analysis of Solitonic Structures in Systems with Higher-Order Dispersion and Nonlinearity. European Conference on Computational Mechanics - ECCM'99, Aug. 31 - Sept. 3, Munich, Germany.

4. A.Ravasoo. Nonlinear longitudinal waves interaction for inhomogeneously predeformed medium characterization. $15^{th}$ International Syposium on Nonlinear Acoustics. 1 - 4 Sept., 1999, Göttingen, Germany.

5. A.Berezovski, G.A.Maugin. "Material Formulation of Finite-strain Thermoelasticity and Applications". Third International Conference of Thermal Stresses "THERMAL STRESSES '99", Cracow, Poland, June 13 - 17, 1999, (General Lecture).

6. A.Berezovski, G.A.Maugin. "Application of Wave-Propagation Algorithm to 2D Thermoelastic Wave Propagation in Inhomogeneous Media. International Conference to honour S.K.Godunov, "Godunov Methods: Theory and Applications", Oxford, October 18 - 22, 1999.

7. A.Braunbrück, A.Ravasoo. Symbolic software application for modelling of wave interaction process. $12^{th}$ Nordic Seminar on Computational Mechanics (NSCM-12). Helsinki, HUT, 1999.

8. J.Kalda. "New lattice model of Brownian surfaces". Scientia Europæa No. 4, 19 - 23 Sep. 1999, Bischoffsheim, France.

9. J.Kalda, J.Engelbrecht. "Mathematics and Medicine". Tallinn Central Hospital Conference, 26. Nov. 1999.

10. J.Kalda, M.Säkki, M.Vainu, M.Laan. 11th Nordic-Baltic Conference on Biomedical Engineering, Tallinn, 6 - 10 June, 1999 (invited sectional lecture).

11. O.Kongas, M.Vendelin, V.Saks. 11th Nordic-Baltic Conference on Biomedical Engineering, Tallinn, 6 - 10 June, 1999 (invited key lecture).

12. M.Vendelin, P.Bovendeerd, T.Arts, J.Engelbrecht, D.H. van Campen. 11th Nordic-Baltic Conference on Biomedical Engineering, Tallinn, 6 - 10 June, 1999 (invited sectional lecture).

13. J.Lass, K.Meigas, T.Lipping. 11th Nordic-Baltic Conference on Biomedical Engineering, Tallinn, 6 - 10 June, 1999 (invited lectures).

14. H.Hinrikus, J.Lass, K.Meigas, J.Riipulk, and V.Tuulik. VI European Conference on Medical Physics, Patras, 1 - 4 Sept., 1999.

15. H.Hinrikus. Low-level microwave field effects on living system. 11th Nordic-Baltic Conference on Biomedical Engineering, Tallinn, 6 - 10 June, 1999 (invited key lecture).

16. H.Aben, L.Ainola, J.Anton. GESA-Symposium "Anspruch und Tendenzen der experimentellen Strukrurmechanik", Warnemünde, 6 - 7 May, 1999.

17. H.Aben, L.Ainola, J.Anton. International Conference on Advanced Technology in Experimental mechanics (ATEM'99), Ube, Japan, 21 - 24 July, 1999.

18. M.Rahula. Int. Conf. Invariant Methods for Mathematical Physics and Geometrical Structures, Moscow, 25 - 30 Oct., 1999.

19. M.Rahula. Ü.Lumiste, Radu Rosca Int. Conf., Brussels-Leuven, 11 -16 Dec., 1999.

20. V.Abramov. Int. Conf. on Dirac operators, index theorems and numerical invariants of manifolds. Greifswald, Germany, 7 - 13 March, 1999.

21. V.Abramov. Sixth Int. Wigner Symp., Istanbul, Turkey, 16 - 22 Aug., 1999.

### 5.4 Seminars outside the home Institute

1. M.Vendelin. "Cardiac muscle: Controversy between cross-bridge model predictions and experiment", Amsterdam, Free University, Dec., 1999.

2. M.Vendelin. "Composing the model of cardiac muscle mechanics and energetics", Eindhoven, Eindhoven University of Technology, Dec., 1999.

3. P.Peterson. "Looking down on waves", Maritime Research Institute Netherlands, Dec., 1999.

4. P.Peterson. "Wave crests", University of Twente, Dec., 1999.

5. H.Aben "Tomographic methods in experimental mechanics", University of Naples, May, 1999.

6. H.Hinrikus. "Microwave effects on human EEG". Lund University, Nov., 1999.

### 5.5 Supportive grants (travel, etc.)

1. German Science Foundation Grant for A.Ravasoo participation in $15^{th}$ International Symposium on Nonlinear Acoustics. 1-3 Sept. 1999, Göttingen, Germany.

2. ETF travel grant: M.Vendelin.

3. ETF young researchers grant: M.Vendelin, P.Peterson.

4. PhD students support: M.Vendelin, P.Peterson.

5. A.Salupere. German Research Association grant to cover the ECCM'99 conference (Munich, Germany).

6. J.Kalda. French Academy of Sciences, grant to attend Scientia Europæ N 4, Bischoffsheim, France.

7. J.Engelbrecht, Exchange Fund of Czech Academy of Sciences to attend EURODYN 99, Prague.

### 5.6 International cooperation

1. Nonlinear dynamics of heterogeneous solids with microstructure. Cooperation between RWTH Aachen and Institute of Cybernetics (see grant N 1).

2. Nonlinear waves in complex media. Cooperation agreement between Laboratory of Mechanics, University of Paris 6 and Institute of Cybernetics.

3. Nonlinear dynamics of biological structures. Cooperation agreement between Stevin Centre, Eindhoven University of Technology and Institute of Cybernetics.

4. Thermodynamics of complex systems. Cooperation agreement between HAS-TUB Research Group for Continuum Mechanics, Hungarian Academy of Sciences, Budapest and Institute of Cybernetics.

5. Nonlinear waves and oscillations. Cooperation agreement between Laboratory of Theoretical and Applied Mechanics, Helsinki University of Technology.

6. Technical University of Darmstadt, Faculty of Building - co-operation on residual stress measurement in thick glass plates used in buildings.

7. Politecnico di Bari, Construction of an automatic polariscope for stress measurement in axisymmetric glass articles.

8. "Verrerie Cristallerie d'Arques", stress measurement methods for glassware.

9. Nonlinear problems in quantum field theory and dynamical systems. Cooperation Agreement between University of Paris 6, University of Wroclaw and University of Tartu.

### 5.7 Research programmes (national)

1. Nonlinear Dynamics (part of the Estonian Programme on Mechanics).

2. Mathematical Modelling of Physiological Processes (Part of the Estonian Programme on Biomedical Engineering).

3. Research programme of Tallinn Piano Factory and Institute of Cybernetics.

### 5.8 Teaching activities

1. J.Kalda: Preparation of the Estonian team of five students to the XXX International Physics Olympiad in Padova (Italy), 18.-26. July 1999. The Estonian team won one gold, one silver, one bronze medal, and two honorably mentions.

2. J.Engelbrecht: Mathematical modelling, TTU, Spring; 1999.

3. J. Engelbrecht, J.Kalda (A.Braunbrück, M.Säkki - assistants): Nonlinear Dynamics and Chaos, TTU, Autumn, 1999.

4. A.Salupere: Undergraduate courses at Tallinn TU:
   "Dynamics" (together with P.Peterson);
   "Statics";
   "Continuum mechanics";
   "Technical mechanics";
   "Theory of elasticity".

5. H.Aben, course "Hybrid and tomographic method in experimental mechanics", Politecniko di Bari, May, 1999.

6. M.Rahula: graduate courses at TU: Lie differentiation and catastrophes, Geometry in contemporary mathematics.

7. A.Parring: graduate course at TU: Noneucleidean geometries.

8. R.Riives: graduate course at TU: Global analysis.
9. V.Abramov: graduate courses at TU: Supergeometry and Supersymmetries Seiberg-Witten invariants.
10. H.Hinrikus: Biological effects of electromagnetic radiation. TTU, Spring 1999.
11. T.Lipping: Physiological signal processing. TTU, autumn 1999.
12. K.Meigas: Biomedical instrumentation. TTU, spring 1999.
13. H.Hinrikus: Electromagnetic fields and waves. TTU, autumn 1999.
14. J.Riipulk. Medical Imaging. TTU, autumn 1999.
15. K.Meigas: Microwave and optical engineering. Autumn 1999.
16. T.Lipping: Signal processing. TTU, spring 1999.

### 5.9 Conferences organized

1. Seminar in Estonian Academy of Sciences, "Order and Chaos", October, 1999.
2. 11th Nordic-Baltic Conference on Biomedical Engineering. IEEE EMBS, EMBE, IOMP Regional meeting, Tallinn, June, 1999 (Conference Chair - H.Hinrikus).
3. International seminar "Contemporary differential geometry and applications", Tartu, June, 1999.

### 5.10 Visiting scholars

1. Dr. Peter Beda, Technical University Budapest. Seminar on "HAS-TUB Research Group of the Dynamics of Machines and Vehicles".
2. Dr. Andras Szekeres, Technical University of Budapest. Seminar on "Application of Reciprocal Relations in Thermo-Hygro-Mechanics".
3. Prof. E. van Groesen, University of Twente, the Netherlands. Seminar on "Unstable bi-chromatic wave evolution: experiments, numerics, modelling & analysis".
4. Prof. Metin Akay, Dartmouth College Hanover, USA. Intensive course "Physiological Signal Processing".

### 5.11 Theses

1. T.Sillat. Wave propagation in dissipative microstructured materials, MSc thesis, Tallinn, TTU, 1999 (supervisor J.Engelbrecht).
2. N.Ilves. Calcium dynamics in myocardial cell, BSc thesis, Tallinn, TTU, 1999 (supervisor J.Engelbrecht).
3. A.Braunbrück. Nonlinear wave propagation and interaction in homogeneous elastic medium, BSc thesis, Tallinn, TTU, 1999 (supervisor A.Ravasoo).
4. M.Säkki. The method of nonlinear dynamics in the analysis of heart rate variability for children, BSc thesis, Tartu, TU, 1999, (supervisor J.Kalda).

5. O.Ilison. Higher-order dispersive effects on solitary waves. BSc thesis, Tallinn, TTU, 1999 (supervisor A.Salupere).

### 5.12 Distinctions

J. Engelbrecht - Dr.h.c. from Budapest Technical University;

N. Ilves - Student award, Estonian Acad. Sci.;

A. Braunbrück - Student award, Estonian Acad. Sci..

J. Lass - Ragnar Granit Prize for Young Scientists at 11th Nordic - Baltic Conf. on Biomedical Engineering.

## 6. Summary

### 6.1 Conclusions and Prospects

In order to enhance the cooperation between the teams working in nonlinear sciences the Centre for Nonlinear Studies (CENS) was created. This report is the first to reflect the concerted activities. An annual seminar of CENS is planned to be organized in February - March 2000 to discuss the results and plan further activities.

The computing facilities have been enhanced and our cluster functions like a whole with parallel processing.

Several theses were finished and young people continue actively their studies combined with research.

The international cooperation with our partners is continuously improving, as well as the cooperation within Estonia.

The ideas put forward to be realized in 1999 are all completed: CENS has been founded, research results were good and more students were engaged in projects.

We have to work in 2000 under a certain cut-off of funding due to the general cut-off the Science Budget in Estonia. Nevertheless, we keep all our activities going. The list of hot problems is given in Section 2.

### 6.2 A view ahead: invited and/or accepted presentations for 2000

1. J.Engelbrecht. Nonlinear solitary waves in highly dispersive microstructured media. Workshop on Nonlinear Waves Solids, Hongkong, June 8-11, 2000.

2. J.Kalda. The methods of non-linear dynamics in the analysis of heart rate variability. 22nd Int. Conference on the Unity of Sciences, Seoul, Febr. 9-13, 2000.

3. J.Kalda. On the multifractal properties of passively convected scalar fields. Fractals 2000, Singapore, April 14-19, 2000.

4. M.Vendelin. Cardiac mechanoenergetics replicated by cross-bridge model. 44th Annual Meeting of Biophysical Society, New Orleans, Febr. 12-16, 2000.

5. O.Kongas. In silico regulation of oxidative phosphorylation in muscle cells in health and disease. Ibid.

6. T.Soomere, K.Rannat. Double monopoles in a laboratory experiment. EGS General Assembly, Nice, 2000.

7. T.Soomere, K.Kasemets. Cyclones with anticyclonic kernel on a beta plane. Ibid.
8. A.Salupere. Technics for detection of solitons in numerical experiments and virtual soliton concept. ECCOMAS 2000, Barcelona, Sept. 11-14, 2000.
9. H.Aben, L.Ainola, J.Anton. Integrated photoelasticity for nondestructive residual stress measurement in glass. Int. Conf. Trends in Optical Nondestructive Testing, Lugano, May 3-5, 2000.
10. M.Säkki. On the nonlinear measures of the heart rate variability of children. EURO-ATTRACTOR 2000, Warsaw, June 6-15, 2000.